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Standard Model of Particle Physics and String Theory

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ABSTRACT

Quantum Physics remains an unapproachable and intimidating topic for high schoolers. Research papers available online contain complicated mathematics, rendering topics such as Quantum Field Theories in the Standard Model and String Theory inaccessible for students trying to gauge interest in these fields.

This paper aims to provide an intuitive understanding, at the least, of these topics. It contains concise, relatively detailed explanations of the quantum field theories that explain the presence and interaction of the fundamental forces in our universe. It explores first the Standard Model, familiarising readers with QED, QCD and Electroweak theory. The study examines flaws in this model before describing key attempts at the unification of forces in the quantum world: Grand Unification theory, Kaluza-Klein Idea and Supersymmetry. It links then to String Theory.

The paper tries an interdisciplinary approach, by examining the implication of String Theory from the perspective of physics and mathematics. Conclusively, more research is required before one can determine whether String theory is realistic or not.

KEYWORDS: *Quantum Physics, String Theory, Standard Model, Gravity, Electroweak Theory, Quantum Electrodynamics, Quantum Chromodynamics, Supersymmetry*

1. STANDARD MODEL

Introduction

Since forever, scientists have been hunting to answer the Big Questions, like “What is the universe made of?”. This Big Question is precisely what the Standard model aims to answer. This model is the most well-founded theory, as of date, to explain the basic building blocks of the universe- the subatomic particles. It explains leptons which form matter, and bosons which are the fundamental carriers of force. Millions of small bosons together form forces of electromagnetism, gravity, and others.

This model, a collection of several distinct theories targeting different forces and phenomena, aims to explain subatomic behavior and the interaction of particles. It classifies all elementary particles, as can be seen in the Standard Model Map below. More than anything, it aims to unify the fundamental forces,

i) Electromagnetism,
ii)Weak force
iii)Strong force
iv) Gravity

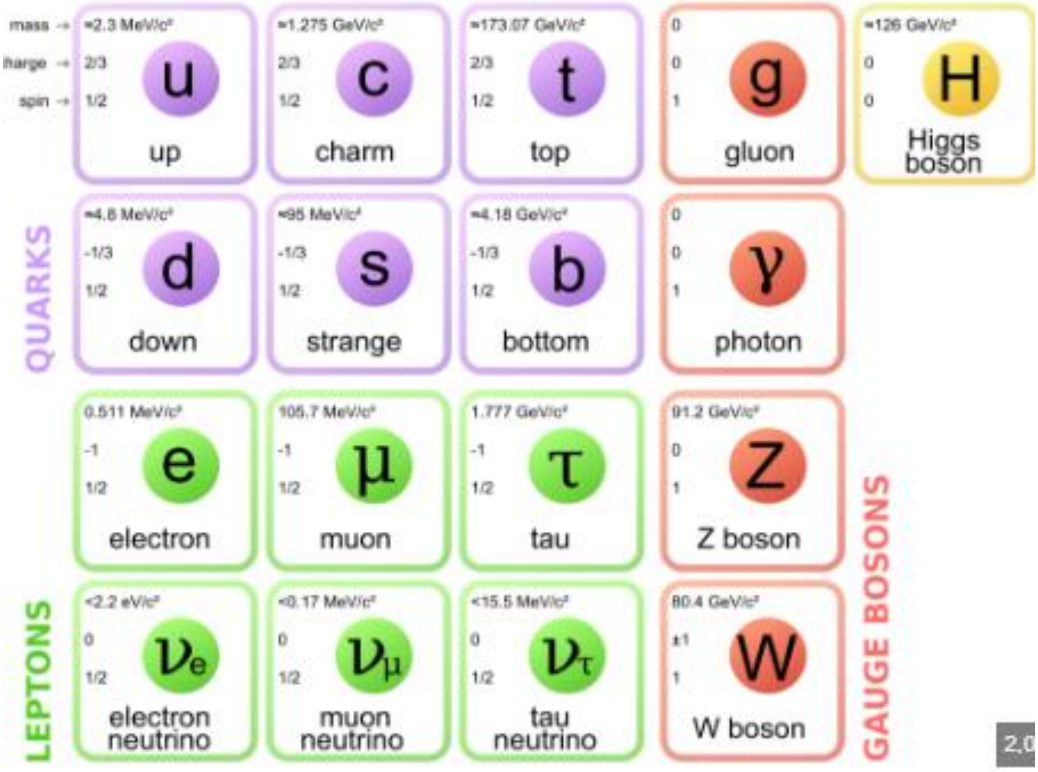


Figure 1: Standard Model

Explaining forces:

i. Electromagnetic force:

This force is explained by Quantum Electrodynamics, one of the most precise theories devised by man. This theory is the QFT (Quantum Field Theory) of the interaction of charged particles with electromagnetic fields. It mathematically explains interactions of light with matter, and of charged particles with each other. It combines quantum mechanics and Einstein's theory of special relativity hence making it successful.

Noether's Theorem can explain this force. Since electric current is conserved, there must be some symmetry. Since there is no physical transformation, the symmetry is internal. This means, we can build in equations into a theory, such that they lead to current conservation.

Since the quantum field has no physical variation, the spacetime in the real dimension does not change. We can consider the quantum field to exist in multiple spacetimes, in internal spaces- the spaces to which we make transformations to build internal symmetry.

All electromagnetism is brought about by charged particles, particularly protons and electrons. Both are fermions. It is possible to create symmetry here if each fermion is paired with its conjugate fermion. (A conjugate fermion would work to cancel the spin and momentum of the original).

Figure 2: (a)Rotation in the anti-clockwise direction of fermion, (b) Rotation of conjugate fermion in the clockwise direction

The two rotations of conjugate fermions cancel, giving symmetry.

However locally, this would cause problems since terms then do not cancel. Herein, we add in an extra field, which is the conjugate for all remaining fermions of the real field. This is called the gauge field. This gauge field behaves as a quantized electromagnetic field.

Excitations in this field are called photons. The exchange of these photons creates force. These combined interactions explain electromagnetism and are the QED theory.

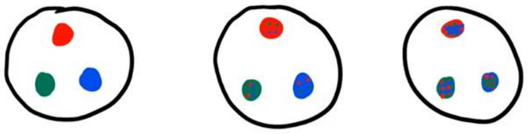
ii. Strong Force:

This force is explained by the theory of Quantum Chromodynamics, or QCD.

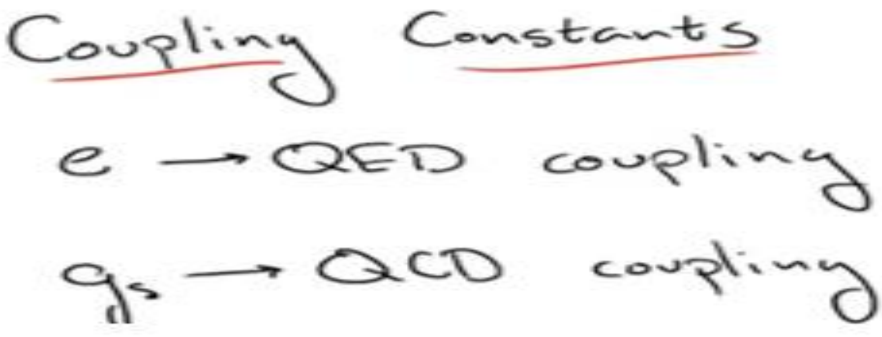
The best way perhaps to explain QCD is by introducing the “chromo” or colorful part of it. This can be achieved with the example of a Δ^{++} baryon.

Figure 3: A Δ^{++} baryon

This baryon contains three up quarks with up spin. This contradicts the Pauli Exclusion.
Principle, which states no state may have more than one fermion with the same spin in the same direction. A state cannot have two electrons with up spin. This means, there exists an additional quantum number that takes other three different values, called colour charge. Its possible values are red, green, and blue.
If the three colour states are orthogonal, the baryon exists. Orthogonal means that the quarks must be perpendicular to each other. Otherwise, the components of their momenta, spins, and other characters may add up or cancel to contradict the Pauli Exclusion Principle. Hadron states are symmetric under colour transformations, so different baryons are not observed for different quark states.



All these states are identical.
In case of fermions, they are complex spinors as stated before, they exist in a complex 3-D vector space. The colour states are normalized, they cannot change their magnitudes. So, the only possible transformations are rotations in the 3-D space.
The collection of these 3-D transformations is called SU(3). SU stands for Special Unitary. It represents the properties of the transformations, that its determinant is 1, and a transformation and its conjugates are inverses. The 3 represents that 3-D is the simplest space transformation (U) can act on.
For hadrons to be unchanged under new interactions, there must be a force opposing the repulsive electrostatic forces between protons. This can be accomplished if the color states could somehow act as charges under interactions since quarks would feel their effect, but hadrons would not.
This can be done by introducing a new field which would cancel all local transformations and introduce symmetry. This field is made up of units-gluons, which act as mediators between color states just as photons-mediated electromagnetic force.
In the case of QED, photons were neutral since leptons and anti-leptons balanced charges. But here, each quark carries a fundamental charge and an anti-fundamental charge. Hence the gluon must be a combination of fundamental and anti-fundamental SU(3) combination. It is seen that an octet or adjoint structure of gluon is most stable here, though its details are not primary to this research.
Since the gluons too are charged, they interact with other quarks as well as each other. The theory explaining their interactions with each other, and other quarks is called QCD.



The fact that this strong force is stronger than EM force is proven using coupling constants.
For QED, the strength of coupling increases with energy increase. The energy of the particles here corresponds to shorter lengths, so the closer two charged particles are, the greater their energy will be.
For QCD, gluons due to their self-interactions, result in greater coupling at lower energy levels. Hence at the same energy level, QCD has greater coupling than QED. This proves strong force is stronger than EM force.

ii. Weak Force:

The concept of weak interactions only explains this force, it does not have a specific theory name, unlike QED and QCD. Initially, let us observe the problem with beta-decay. A proton can convert to a neutron, which happens to be slightly heavier than the proton. This clearly violates the law of conservation of mass, hence energy for relativistic particles.

Let us momentarily consider the muon, a particle with all properties but mass, identical to the electron. It cannot undergo strong interactions and has no isospin properties.

A muon decay looks like this:

It is important to note the products: electron, antineutrino, and neutrino for a μ-.

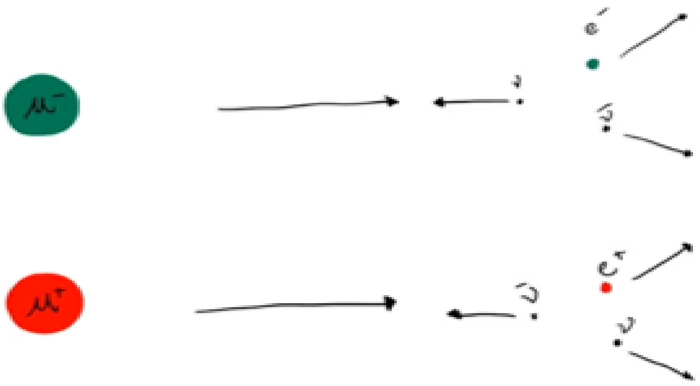


Figure 4: μ^+ and μ^- decay

A beta decay here would look like:

Here there is an electron and neutrino for a proton, only 2 particles.

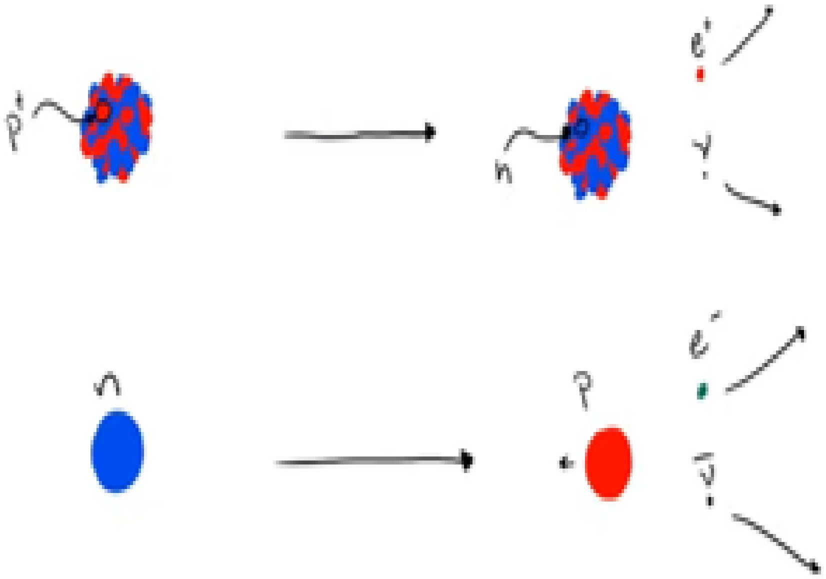


Figure 5: β decay

The issue with this type of beta or weak decay is that since there are no electric or colour charges, we have no obvious way of converting one type of particle into another since in QED and QCD it happened using these quantum numbers. The earliest idea was to add new interactions to Feynman diagrams to represent neutrinos. It would be illogical to simply add new interaction. Further, QED and QCD were on the basis of symmetries. These new interactions would also not be renormalizable, making calculations difficult.

There was another way, based on Heisenberg’s uncertainty principle, which states one cannot know the position and velocity of a particle accurately at the same time. This can be applied to time and energy as well. Its implication is that a large amount of energy can be taken from the surroundings, provided it is returned in a short interval of time.

In a usual interaction, there would be a massless photon between the incoming and outgoing particles. Let us consider a particle Z with a large mass M(z).

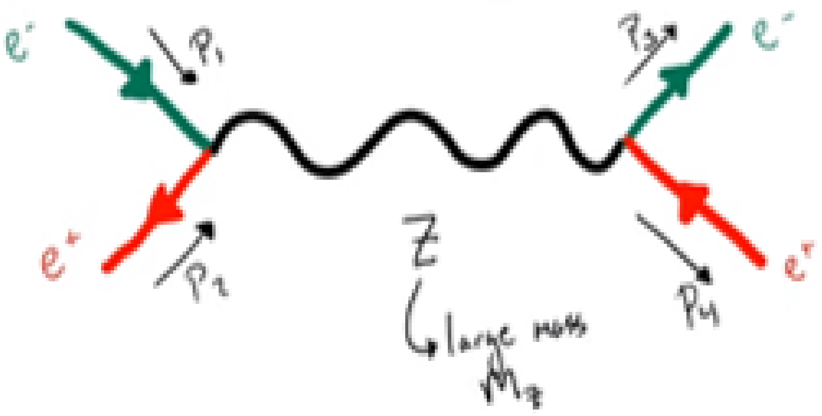


Figure 6: Exchange of energy between M(z) and surroundings

This exchange would not be possible generally since M(z) would be more than energies of incoming particles, and energy conservation would not occur. But by the Heisenberg principle, the system can borrow energy from its surroundings, at the cost of the time it retains energy.

This heavy intermediate particle is the Z boson, and the incoming-outgoing interacting ones are W^+ and W^- bosons. These interactions are called weak charged-current interactions and renormalize the process. However, it was unsatisfactory since photons and gluons were massless, and originated from symmetries.

iii. Electroweak Theory

This theory unifies the EM and weak forces.

In point iii, we discussed how the lack of gauge symmetries made the explanation for weak force unsatisfactory. The reason, as mentioned, was that W^+ bosons are massive, and symmetry is broken when mass is introduced. This can be avoided by giving gauge bosons mass by spontaneously breaking their symmetry. This means W^+ boson could arise from a gauge boson after Higgs' Mechanism runs its course.

Let us recall two facts:

i). Gauge bosons self-interact and they may be charged.

ii). When gauge bosons lose symmetry by SSB (spontaneous symmetry breaking), it is possible that only some are made to lose symmetry and others are made to retain it.

Now, the gauge bosons which gained mass can self-interact with massless ones, and remain charged under gauge symmetry. This is analogous to how W^+ bosons are charged under QED.

Parity comes in here, the property by which spatial systems are reflected. Fermions, or complex spinors, are of two kinds- Dirac(4 components) and Weyl(2 components).

Dirac = Left-handed Weyl + Right-handed Weyl. This means a Dirac made from two Weyl spinors would violate parity since it is no longer symmetric about reflection.

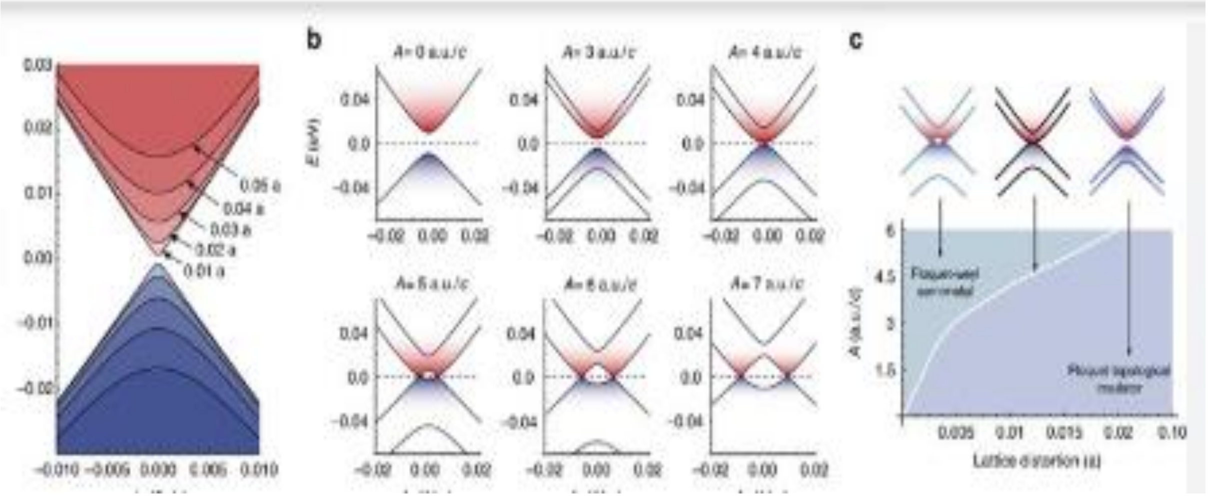


Figure 7: A Weyl semimetal image, computer diagrammatic hologram of a possible Weyl fermion

This is important to note, since it suggests the use of Weyl spinors for theories that violate parity. Further, weak decays violate parity, they treat left-handed and right-handed fermions differently- the W^+ and W^- bosons interact only with left-handed fermions. For unification hence, the interaction must only interact with left-handed fermions. This proposition is the Chiral Gauge Theory.

To achieve unification, we need an i) gauge symmetry which can be spontaneously broken by a scalar, ii). Some preserved symmetries after symmetry breaking, iii) self-interactions for charged W^+ and W^- bosons, iv) chirality of interaction, v) fermions paired according to weak decay (down with up and such), vi) interaction of fermions with the scalar.

For the unification to work, there must be at least 3 charged bosons that must exist after symmetry breaking, one to form a photon and the other two for $w^{+/-}$ bosons. For this interaction, we need an SU(2) gauge symmetry, containing two complex-dimensions. This is important since QED contains complex internal rotations, hence the symmetry cannot occur in a real space. The fermions are then arranged in pairs according to their weak interaction characteristics and then transformed in an SU(2) gauge symmetry. This complex transformation gives rise to QED since it uses the particles of weak interactions to give rise to electromagnetic force. When this happens all charged leptons also get paired with neutrinos. However, here we have only assumed the case for left-handed fermions.

The criteria of Dirac masses of fermions and parity symmetry for QED are now examined. We need a scalar for this, which will break this SU (2) symmetry to give fermions having mass, but it is not possible for a scalar to exist, which will break an SU (2) symmetry to give chirality while also giving fermions mass. For this, we must consider the addition of right-handed fermions into the mix for all left-handed fermions present after symmetry breaking. Here, we only add RH fermions and not neutrinos as well, since they are uncharged and do not play a part here hence their mass is not seen in the Dirac equation, producing a claim, neutrinos and whether they are massless. (see point 1 under limitations, SM).

Since weak forces arise from parity breaking and QED is parity-symmetric, SU (2) cannot interact with right-handed fermions. There is another interaction introduced now, which interacts with both the fermions, giving rise to QCD.

Interaction 2 → QED

SU (2) → Weak Force

We let this Interaction 2 be only a U (1) a 1D complex rotation system. It acts on complex objects and transforms them only in one dimension.

Under SU (2) the interaction has a weak isospin charge and under U (1) a weak hypercharge, this combined symmetry can be broken by a scalar to give a Higgs Doublet which has a non-zero weak hypercharge.

$$SU(2)_L \times U(1)_Y \xrightarrow[\text{vev}]{\text{H obtains}} U(1)_{em}$$

After symmetry breaking, we get a parity-symmetric U (1) gauge symmetry, which can now give rise to QED.

With this, both weak and EM forces can arise from these interactions, and unification has been proven.

This is all about the electroweak theory that pertains to this paper. Further consideration into the fate of the four fermions, or rules of scalars is not required.

iv. Elementary particles classification

Too many particles with different spins and masses were discovered in that time. Hence different particles with similar masses, spins, and parities were grouped based on another property isospin. It is a quantum number used to group different hadrons based on similar mass, spin, parity and others. The fundamental units of isospin are up and down spins, up represented as ‘u’ with +1/2 and down as ‘d’ with -1/2.

Isospin also explains why a strong force is stronger than a weak force. Hadrons, classified using isospin, can undergo strong or weak decay. Isospin is conserved in strong decay only; hence it is preferred and stronger. These hadrons will decay with strong decay until they form pions, which are light enough to undergo weak decay until stable.

v. Flavors

When the muon was discovered, it was observed that the electron and muon had very similar interactions.

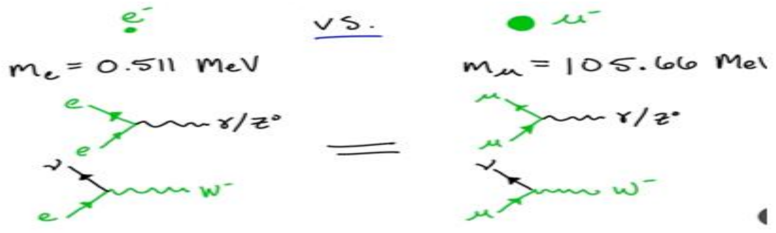
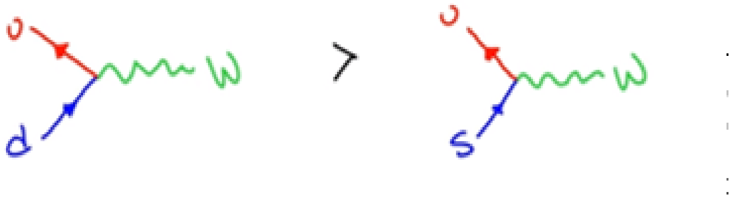
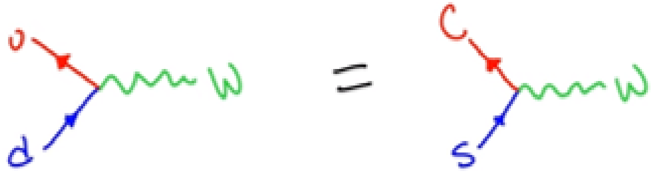


Figure 8: μ and e interactions

This is Lepton Flavor Universality, multiple leptons with no difference but mass. This would suggest that any other leptons discovered would also show similar character, they would be produced in the same way as muons, decay on interaction with W^-bosons, and even disintegrate into both electrons and muons. This was seen, and this introduced the τ electron, and its corresponding τ-neutrino since weak decays only happen between an electron and its corresponding neutrino.

Kaons: spin-zero particles with large masses. Kaons are produced by pions and protons colliding rapidly, which would suggest that kaons could undergo strong decay to form pions. Surprisingly, it undergoes a very slow decay to form pions, and it preferred to form muons and muon-neutrino more than pions.
The solution to this perplexity is strangeness, a quantum number conserved by strong interactions but not by weak interactions (isospin does just the opposite!). This is why kaons can only decay weakly but can be produced strongly.
This complication of isospin and strangeness is termed the eightfold way.
Kaons are made of up, strange, down quarks and their antiquarks. It must undergo weak decay till it forms only up,down quarks and their antiquarks, or all strange quarks must decay and disappear from the picture.





Another interesting fact is that weak interactions treat up and down quarks differently as compared to up and strange quarks. The up-strange interaction is way weaker. This brings in quark mixing and the casino matrix.
If C is any quark with the same properties as the up-quark other than the mass, the two interactions could in principle by the universality of lepton flavor. These two states would now exist in superposition since they have the same quantum numbers, and they could thus, mix. This introduces eigenstates and unitary transformations (where multiplying two conjugate quarks would not alter properties but would assist calculation).
Weak Eigenstates, by unitary transformation, could generate interactions between all quarks. There are also Mass Eigenstates. Certain quarks do not have well-defined masses. Mass determines the energies of quarks by $E=mc^2$. These quarks would not have well-defined energy. Unitary transformation allows us to express such a quark in terms of quarks with well-defined masses, and these transformed quarks are mass eigenstates. These unitary transformations will not affect QED, QCD or other theories.
This strangeness led to the discovery of the strange quark and charm quark.

vi. CP violation

Imbalance between matter and antimatter.

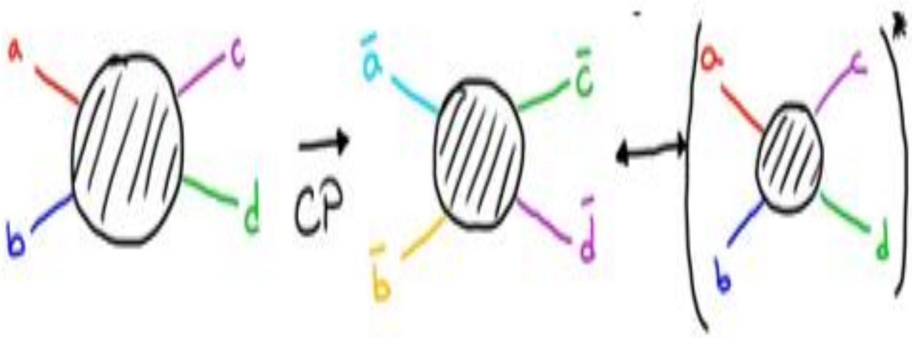
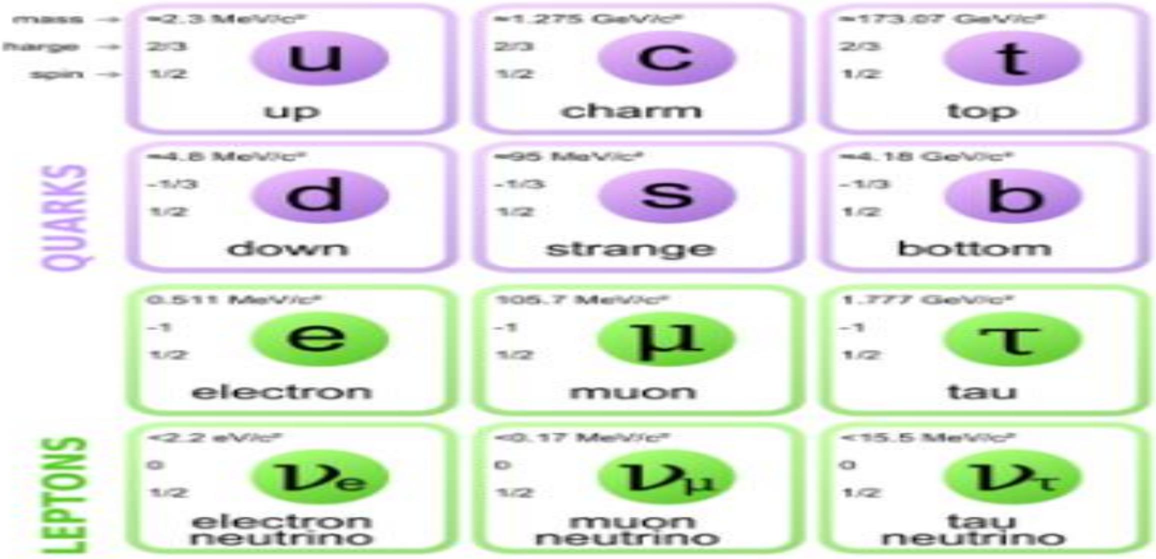


Figure 9: CP Violation

If we take a process and replace all particles with anti-ones, we perform a CP transformation. Since a part of this is complex, CP violation occurs. Such a process cannot occur with complex physical characters, which strongly insist on the existence of more than four quarks. Any CP violation can occur only if more than 6 quarks are present. Further research proved that these quarks were top and bottom quarks. The details of that part are not significant to this paper.
Hence, the entire part of the Standard Model shown below can be understood from points v,vi,vii explaining more than two-thirds of

the model. The rest are formed from bosons, discussed individually in the force explanations.



2. Limitations of the Standard Model:

Gravity:

The particle for gravity, graviton was explained in the Standard Model. SM uses gauge theories to explain interactions of photons, gluons, and bosons- all particles having a spin of magnitude 1. Graviton, however, has spin 2. Gauge theories only cater to particles with spin 1 and cannot hence, explain gravity. A corresponding quantum theory to explain graviton, and subsequently, gravity could not be incorporated into the SM even yet. Since the aim of SM is the unification of forces, this is a serious drawback.

Neutrino-mass problem:

The SM describes the neutrinos to be massless (see Electroweak Theory). However, the three types of neutrinos can transform from one to another depending on how fast they move. As their energy increases, they change form. Energy relates to mass by $E=mc^2$, hence the neutrinos must have mass. This suggests that a part of the Standard Model needs to be revised.

Dark matter:

Dark matter accounts for nearly 95% of the universe. This is matter we cannot see. Scientists uncovered its existence when they realised that galaxies were spinning too fast for the gravitational pull of visible matter. They concluded there must be some invisible matter providing extra mass and increasing gravity. The Standard Model cannot explain dark matter and dark energy. However, scientists believe that the Standard Model is still majorly right and must be a part of a larger correct theory explaining dark matter.

Unification of forces:

The Standard Model does not explain gravity; hence it cannot unify all four forces to give an equation describing the entire universe (the aim of quantum theories). Even among the three described forces, it could only explain the unification of electromagnetism and weak force. It largely prefers to treat forces as singular entities.

Assuming Forces:

In the Standard Model, we assume that forces exist, and we try to reason why. We hope for a single theory form in which forces would arise by themselves, instead of trying to fit forces into a theory.

Attempts at Unification:

Grand Unification: Grand unification theory revolves around the idea that the three fundamental forces having spin 1 are governed by gauge theories. It is possible to consider then, that there may be a single gauge force from which these forces arise at different frequencies. This suggests that at high frequencies, the three forces exist as the same force- a gauge force, and gluons, bosons and photons are all symmetrical to each other in that case, under a gauge symmetry. Operating at lower frequencies, breaks this symmetry, and the force breaks into smaller counterparts. Each counterpart corresponds to one fundamental force, operating at a particular frequency.

This idea is like what we explored in Electroweak theory. The problem here, is that this unification would occur at energy level with orders of 10. This is a very high energy level, which we cannot access yet, so hypothesis verification is not possible. Further, since mass and energy for fundamental relativistic particles are interrelated by Einstein's $E=mc^2$, this implies much higher masses for fundamental particles than what is known. This raises a problem, known as the mass discrepancy problem.

Kaluza-Klein Idea: This idea suggested that instead of three dimensions, there were four spatial dimensions. It was claimed that in this scenario only gravitation existed, no EM. The fourth spatial dimension considered would be so compacted, that these four could be considered as three. Further, the fourth dimension was unidirectional- one could only move in one way in that dimension.

Calculations show, here the graviton with spin2 would be broken into a spin2 and spin1 particle. Further calculations, derivations, and proofs provided that unification would be a possibility, but details of that are irrelevant to us.
The problem was the large number of spatial dimensions. The theory was proposed in the early 20th century when gravity had not been explained even in three dimensions (as is still the case!). There was no quantum theory to support this hypothesis since the theory to support such an argument was only made by Einstein with his Special Relativity, at a much later date.

Supersymmetry: Supersymmetry stemmed from the idea that fermions and bosons might be symmetric to each other in some way and could be linked. After this was disproven, an idea arose that known bosons and fermions might be symmetric to unknown bosons and fermions. Since symmetry would imply the same mass, this suggests pairs of bosons and fermions having the same mass. Since known particles all have different masses, it means the symmetry was broken at some energy scale. Above this scale, this symmetry exists.
Ideas of grand unification and supersymmetry were combined. It was hypothesized that grand unification occurred at very large magnitudes, but supersymmetry at a much lower energy value, just a little above what our accelerators currently operate. Since symmetry is achieved earlier when grand unification occurs, the predicted mass of particles decreases greatly and is nearly adjustable with the known masses, solving mass discrepancy.
There also exists coupling constants (not required for now), one for each fundamental force. GU suggests that these must be unified too, but it was not happening at any energy level. By considering supersymmetry, an energy level where unification occurs has been found.

String Theory

String theory is another take on unification. What makes it interesting, is the notion that the fundamental unit of the universe is perhaps not a particle at all, but a one-dimensional string.

These strings can be opened or looped.
Since these strings are being considered in the place of particles, it is certain that some analogies exist. For the mass parameter of particles, we have frequencies and tensions here.

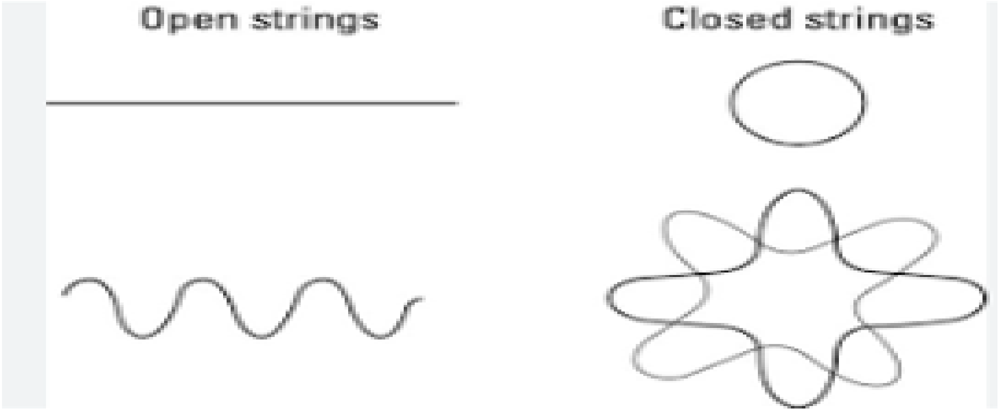


Figure 10: Open Strings and Closed Strings

Each string, like a normal classical three-dimensional string, can oscillate with different frequencies. As the frequency it oscillates with changes, the amplitude will change and the vibration seen will be different that seen initially. As one string vibrates, the quantum strings in its surroundings, up to a certain distance, start to oscillate with the same oscillation mode. At a particular mode, these bundles of waves appear to demonstrate characteristics of an elementary particle with a specific mass. At a different oscillation mode, the same string would appear as a different fundamental particle.

This theory suggests just one type of string as opposed to the numerous fundamental particles that comprise the SM.

A string is made of infinitely many point particles. While constructing experiments or hypotheses then, it had to be considered that such a string would have infinitely many degrees of freedom- the number of independent variables- and working out the problem suggested that a string exists in 26 dimensions including time. It hints at the possibility of 21 hidden dimensions, perhaps which like Kaluza-Klein idea, have been extremely compactified.

What is interesting to note here, is that mathematics has worked out a certain number of spacetime dimensions for us in this theory. In previous ideas, we have always made assumptions on the number of dimensions as four.

The predicted particle-like excitations of this theory indicate the presence of a spin2 particle which can become massless. We have established in this paper earlier, that this energy-mass interlinking allows to particle to maintain a constant energy value and travel large distances due to masslessness. Since it can transverse large distances, it becomes effective over long distances, properties which all add up with those of the graviton. Thus, the prediction of an exact particle that could fit the description of the infamous graviton is what makes string theory a potential mine.

However, string theory also predicts a particle-like excitation, the tachyon, whose mass is an imaginary number. This causes us to consider if tachyon is an unphysical object. Research on it continues.

Successes

Predicts the graviton: This has not been achieved by other theories like SM.

Removes singularities: The main problem with considering fundamental units as particles was the development of singularities and the idea that there existed a definite point at which the graviton would split into two particles. Since strings are extended, the scattering of a string is smooth, removing singularities and other inconsistencies from the picture.

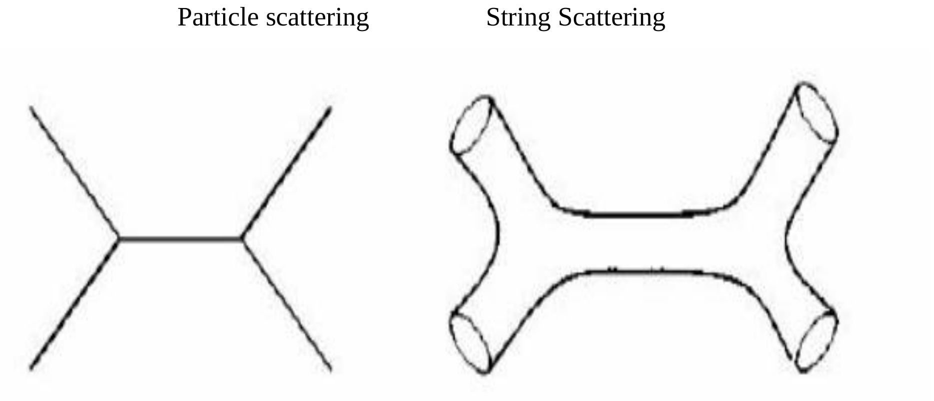


Figure 11: (a)Particle Scattering, (b)String Scattering

Drawbacks

Spatial Dimensions: It predicts 26 spatial dimensions. This makes the mathematics difficult to replicate and work with.

No experiments: String theory is only a theory, there has been no experimental verification that has been successful.

Tachyon: It predicts a particle excitation, the tachyon, which has an imaginary mass.

Superstring Theory:

This theory combines string theory and supersymmetry ideas. The tachyon has an imaginary mass, a square root of a negative number which we can consider as negative energy. The minimum energy of a system tends to be non-vanishing for it to be real. When it is vanishing or has zero-point energy, we get tachyon masses.

If we consider supersymmetry, the zero-energy points of two fermions and two bosons would now cancel each other out, eliminating the tachyon excitation. It also works to reduce spatial dimensions to 10 from 26. The graviton is retained from string theory.

Drawbacks

1. No experimental verification.
2. Still a large number of spatial dimensions
3. Complicated equations which are difficult to work with

There are multiple types of supersymmetries, again their details do not pertain to this study.

Use in Mathematics:

One of the largest drawbacks of string theory is the lack of correct predictions for the results of experiments, unlike SM which predicted and confirmed various particles. As it turns out, string theory does make predictions, only in the field of math and not physics!

Math involves certain multidimensional fields known as Kohler manifolds, which have complex numbers stitched into their structures.

Finding out how a string moves here could help discover the shape, and subsequently, properties of that manifold. Supersymmetry comes in here, wherein bosons and fermions can interchange places, producing invariants in the field. Some of these invariants produce crucial results in the fields of geometry and algebra. It also serves as a base for mirror symmetry, the idea that changing the equations for the strings minorly could produce mirror Kohler manifolds and allow us to describe otherwise complex ideas such as

Hodge numbers. This adds more validity to the existence of string theory in nature and emphasizes that no conclusion about the string theory can be made yet.

3. CONCLUSION

From this paper, we can conclude that further research is much required. While the Standard Model can very well explain each force besides gravity, it often adds new interactions or predicts the existence of other gauge fields- which is unsatisfactory. It is considered that the correct theory describing all the forces will inadvertently point to their existence, instead of trying to fit them into the theory. With String Theory, the above criteria are met. All the forces emerge from its equations, but it is very far-fetched. Its only true predictions were in the field of maths, which causes us to question if string theory is identical to some abstract form of mathematics we have not stumbled upon yet, or if it is the God Equation- containing theory.

Supersymmetry does offer much hope in this regard. The search for a true unification must continue, it is evident that we are not there yet. All these theories contribute some pieces to the grand unification we seek however, and we cannot do away with even the most far-fetched ones of them yet.

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