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Permutation series and combination series

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ABSTRACT

The Permutation Series and Combination Series will ensure the readers to understand proper calculation of Permutation and Combination and will also help them to grew their knowledge on the topic. It is very helpful topic for the students to strengthen their roots in mathematics. The magic that Mathematics have in it will be exposed to the readers to understand their Mathematics thoroughly and will also help them to enlarge their thinking capabilities. Hence, this topic is very interesting as it provides a link between A.P, G.P with Permutation and Combination. It will help to calculate many selections and others in a very small time.

Keywords— Logic, Series, Algebra

1. INTRODUCTION

Till date we have just used permutations and combinations in arrangements, derangements, selection and others. But here, Permutation and Combination is described in a new form called Permutation series and Combination Series.

2. IMPORTANCE OF PERMUTATION SERIES AND COMBINATION SERIES

Just like A.P, G.P, H.P Permutation series and Combination Series we also help us to find out the next term, previous term, sum of Permutations and Combinations up to m terms. It will also strengthen Permutations and Combinations in the minds of the peoples

This will make a easy way to find larger terms in a very short time interval. Hence, the Permutation series and Combination series is largely important.

3. PERMUTATION SERIES

The formula to find Permutation of n things when r things is taken at a time is:

$$^{n}p_{r}= n!/(n-r)! \qquad (1)$$

but Permutation series deals with more than 1 permutation keeping n fixed.

Let us take an example:

$$^{n}p_{r}+ {}^{n}p_{r1}+ {}^{n}p_{r2}+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+{}^{n}p_{r}$$

In Permutation and Combination Series the last highest term is $^{n}p_{n}$ or $^{n}c_{n}$ as n cannot be less than r.

3.1 To find the next term when any term is given(except the first term)

Let $t_{f}={}^{n}p_{r}$ and $t_{f+1}={}^{n}p_{r+1}$

$$T_{f}/t_{f+1}=n!/(n-r)!/n!/(n-r)!$$
$$T_{f}/t_{f+1}=[n!*(n-r-1)!]/[(n-r)!*n!]$$
$$T_{f}/t_{f+1}=(n-r-1)!/[(n-r)(n-r-1)!]$$
$$T_{f}/t_{f+1}=1/(n-r)$$
$$T_{f+1}/t_{f}=n-r$$

Hence,

$$T_{f+1}=(n-r)*t_{f} \qquad (2)$$

3.2 To find the previous term when any term is given (except the first term).

$$T_{f}={}^{n}p_{r} \text{ and } t_{f-1}={}^{n}p_{r-1}$$
$$T_{f-1}/t_{f}=n!/(n-r+1)!/n!/(n-r)!$$

$$T_{f-1}/t_f=[n!*(n-r)!]/[(n-r+1)(n-r)!*n!]$$
$$T_{f-1}/t_f=1/(n-r+1)$$

Hence,

$$T_{f-1}=t_f/(n-r+1) \quad (3)$$

3.3 To find any term when a term is given and common difference is known(except the first term).

Suppose, $t_{a1}={}^np_{r1}$ and $t_{a2}={}^np_{r2}$

$$d=r_2-r_1$$
$$r_2=d+r_1$$
$$t_{a1}/t_{a2}=n!/(n-r_1)!/n!/(n-r_2)!$$
$$t_{a1}/t_{a2}=[n!*(n-r_2)!]/[(n-r_1)!*n!]$$
$$t_{a1}/t_{a2}=[n-(d+r_1)]!/(n-r_1)!$$

Hence,

⇨ $T_{a2}=[(n-r_1)!/(n-d-r_1)!]*t_{a1}$ ---(4)

Where r_1 can be any number less than n and d is the common difference between r_1, r_2, r_3 and so on.

3.4 To find mth term in a Permutation Series

Suppose, a Permutation Series be

$${}^np_1+{}^np_2+{}^np_3+\ldots\ldots\ldots\ldots\ldots\ldots+{}^np_n$$

find the 3rd term.

Solution: To find the third term.

First let write the details

First term $a_r=1$

Common difference=d=2-1=3-2=1

Hence,

Third term:

$$T_3={}^np_{1+(3-1)1}={}^np_3$$

Hence,

The formula to find mth term in a Permutation Series

$$T_m={}^np_{ar+(m-1)d} \quad (5)$$

Where, a_r is the first term, d is the common difference.

3.5 To find sum of m terms

Suppose, a series be

$${}^np_1+{}^np_2+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+{}^np_n$$

find the sum of first 3 terms:

solution: sum of first 3 terms

$$s_m=n![1/(n-1)!+1/(n-2)!+1/(n-3)!]$$

Hence,

In general, to find sum of m terms the formula

$$S_m=n![1/(n-r)!+1/(n-(r+d))!+\ldots\ldots.+1/\{n-r-(m-1)d\}!] \quad (6)$$

Where s_m means sum of the series.

4. COMBINATION SERIES

The formula to find combination of n things when r things is taken at a time is:

$${}^nc_r=n!/[r!*(n-r)!] \quad (7)$$

Just like Permutation series, Combination Series will have highest terms as ${}^nc_r$ as r cannot be more than n keeping n constant. For example:

$${}^nc_1+{}^nc_2+{}^nc_3+\ldots\ldots\ldots\ldots\ldots\ldots+{}^nc_n$$

4.1 To find the next term when any term is given(except the first term).

Let $t_f={}^nc_r$ and $t_{f+1}={}^nc_{r+1}$

$$T_f/t_{f+1}=n!/[r!(n-r)!]/n!/[(r+1)!*(n-r-1)!]$$
$$T_f/t_{f+1}=[n!*(r+1)!*(n-r-1)!]/[r!*(n-r)!*n!]$$
$$T_f/t_{f+1}=[(r+1)(r!)*(n-r-1)!]/[r!*(n-r)(n-r-1)!]$$
$$T_{f+1}/t_f=(n-r)/(r+1)$$

Hence,

$$T_{f+1}=[(n-r)/(r+1)]*t_f \quad (8)$$

4.2 To find the previous term when any term is given (except the first term)

Let $t_f={}^nc_r$ and $t_{f+1}={}^nc_{r-1}$

$$T_{f-1}/t_f=n!/[(r-1)!*(n-r+1)!]/n!/[r!*(n-r)!]$$
$$T_{f-1}/t_f=[n!*r!*(n-r)!]/[(r-1)!*(n-r+1)!*n!]$$

$$T_{f-1}/t_f = [r(r-1)!*(n-r)!]/[(r-1)!*(n-r+1)(n-r)!]$$

$$T_{f-1}/t_f = r/(n-r+1)$$

Hence,

$$T_{f-1} = [r/(n-r+1)]*t_f \quad (9)$$

4.3 To find any term when a term is given and common difference is known (except the first term)

Let $t_a = {}^nc_{r1}$ and $t'_a = {}^nc_{r2}$

$r_2 = r_1 + d$ where d is common difference and r_1 is any number less than n.

$$t'_a/t_a = [n!*r!*(n-r_1)!]/[r_2!*(n-r_2)!*n!]$$

Hence,

$$t'_a = [\{r_1!*(n-r_1)!\}/\{(r_1+d)!*(n-r_1-d)!\}]*t_a \quad (10)$$

4.4 To find m^th term

Let the first term be ${}^nc_r$

$$a = r$$

Just like Permutation Series, in Combination Series, to find the m^{th} term we use the formula:

$$T_m = {}^nc_{a+(m-1)d} \quad (11)$$

Where d is common difference.

4.5 To find sum of m terms

Just like Permutation Series, if s_m be the sum of m terms of a permutation series and first term is ${}^nc_r$ where a=r then the formula:

$$S_m = n![1/(r!(n-r)!)+1/((r+d)!(n-r-d)!)+......................1/((r+(m-1)d)*(n-r-(m-1)d)] \quad (12)$$

4.6 Some special combination series with their formula for sum:

$${}^nc_0+{}^nc_1+{}^nc_2+\ldots\ldots+{}^nc_n = 2^n$$

$${}^nc_1+{}^nc_2+\ldots\ldots+{}^nc_n = 2^n-1$$

$${}^nc_0-{}^nc_2+{}^nc_4-{}^nc_6+\ldots\ldots = (2\text{^}0.5)^n \cos(n*180/4)$$

$${}^nc_1-{}^nc_3+{}^nc_5-{}^nc_7+\ldots\ldots = (2\text{^}0.5)^n \sin(n*180/4)$$

$${}^nc_0-{}^nc_1+{}^nc_2-{}^nc_3+\ldots\ldots+(-1)^n\,{}^nc_n = 0$$

$${}^nc_1-2\,{}^nc_2+3\,{}^nc_3-\ldots\ldots = 0$$

$${}^nc_0+2\,{}^nc_2+3\,{}^nc_3+\ldots\ldots+(n+1)\,{}^nc_n = (n+2)2^{n-1}$$

$${}^nc_0\,{}^nc_r+{}^nc_1\,{}^nc_{r+1}+\ldots\ldots+{}^nc_{n-r}\,{}^nc_n = {}^{2n}c_{n-r} = (2n)!/[(n-r)!*(n+r)!]$$

$${}^nc_0^2+{}^nc_1^2+{}^nc_2^2+\ldots\ldots+{}^nc_n^2 = (2n)!/(n!)^2$$

$${}^nc_0^2-{}^nc_1^2+{}^nc_2^2-\ldots\ldots+(-1)^n\,{}^nc_n^2 = \{{}_{(-1)\text{^}n/2.ncn/2 \text{ if n is even}}{}^{0 \text{ if n is odd}}\}$$

$${}^nc_1^2-2\,{}^nc_2^2+3\,{}^nc_3^2-\ldots\ldots+(-1)^n\,n.{}^nc_n^2 = (-1)^{n/2-1}n/2[(n!)/\{(n/2)!*(n/2)!\}] \text{ when n is even}$$

$${}^nc_0+{}^nc_1/2+{}^nc_2/3+\ldots\ldots+{}^nc_n/(n+1) = (2^{n+1}-1)/(n+1)$$

$${}^nc_0+{}^nc_1/2+{}^nc_2/2^2+\ldots\ldots+{}^nc_n/2^n = (3/2)^n$$

$${}^{2n+1}c_0+{}^{2n+1}c_1+{}^{2n+1}c_2+\ldots\ldots+{}^{2n+1}c_n = 2^{2n}$$

$${}^nc_n+{}^{n+1}c_n+{}^{n+2}c_n+\ldots\ldots+{}^{2n-1}c_n = {}^{2n}c_{n+1}$$

$$\sum_{r=0}^{n}(-1)^r\,{}^nc_r\{1/2^r+3^r/2^{2r}+7^r/2^{3r}+15^r/2^{4r}+\ldots\ldots+m \text{ terms}\} = (2^{mn}-1)/(2^{mn}(2^n-1))$$

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